Awareness Logic: A Kripke-based Rendition of the Heifetz-Meier-Schipper Model and a Dynamic Extension

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ABSTRACT

Heifetz, Meier & Schipper (HMS) present a lattice model of awareness. The model is syntax-free, precluding the simple option to rely on formal language to induce lattices, and represents uncertainty and unawareness with one entangled construct, making it difficult to assess their properties. We present a model based on a lattice of Kripke models, induced by atom subset inclusion, in which uncertainty and unawareness are separate. We show the models equivalent by defining transforms between them that preserve formula satisfaction, and get completeness through our and HMS' results. Lastly, we show that in Kripke lattices DEL-updates are readily applicable, thereby showing awareness dynamics as well.

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1 INTRODUCTION

Awareness has been intensively studied in logic and game theory since its first formal treatment by [7]. In these fields, uncertainty is considered the agent's ability to distinguish possible states of the world based on its available information, while awareness is the agent's ability to even contemplate aspects of a state.

Heifetz, Meier and Schipper (HMS) propose a syntax-free framework to model awareness [9]. Indeed, in their *unawareness frames,* "atomic" and epistemic events are defined without appeal to atomic propositions or other syntax. The model is based on a lattice of spaces, where uncertainty and unawareness are represented *jointly* by a *possibility correspondence* Π_a for each agent $a \in Ag$. Π_a maps a state to weakly less expressive states, representing agents that may lack full awareness of the mapped-from state.

We find the HMS lattice-based conceptualization of awareness elegant, interesting and intuitive—but we also find its formalization cumbersome. The choice to go fully syntax-free robs the model of the option to rely on formal language to induce lattices and specify events, resulting in constructions which we find not very easy to follow. This may of course be an artifact of us being accustomed to non-syntax-free models used widely in epistemic logic.

Another artifact of our familiarity with epistemic logic models is that we find HMS' joint definition of uncertainty and unawareness difficult to relate to other formalizations of knowledge. When HMS propose properties of Π_a, it is not clear to us what concerns knowledge and what concerns awareness. They merge two dimensions which, to us, would be clearer if left separated.

With these motivations, we propose a non-syntax-free, Kripke model-based rendition of the HMS model [2]. Roughly, we start from a Kripke model K for a set of atoms At, spawn a lattice containing restrictions of K to subsets of At, and add maps π_a on the lattice that take a world to a copy of itself in a restricted model. This keeps the epistemic and awareness dimensions separate: accessibility relations R_a of K encode epistemics while maps π_a encode awareness. We show that when each R_a is an equivalence relation and each π_a satisfies three identified properties, the result is equivalent to the HMS model, as the two satisfy the same formulas of the language of knowledge and awareness.

Going beyond [2], we present here a further advantage of Kripke lattices. As they are based on Kripke models, standard DEL machinery can be readily applied to them [1]. Thus, we show how to introduce action models and product updates in Kripke lattice models, thereby capturing awareness dynamics as well.

Throughout this work, we assume that Ag is a finite, non-empty set of agents, and that At is a countable, non-empty set of atoms.

2 HMS MODELS

HMS models are based on unawareness frames. The backbone of such frames is a complete lattice of state-spaces $(\mathcal{S}, \preceq)$, with the intuition that the higher a space is, the richer the "vocabulary" it has to describe its states. Since the HMS approach is syntax-free, this intuition is not modeled using a formal language. It is represented using $\preceq$ and a family of projections $\mathcal{R} = \{r^{S'}_S : S, S' \in \mathcal{S}, S \preceq S'\}$, where each $r^{S'}_S$ projects state-space S' down to S. Projections are used to define events, which are any pair $(D^{\uparrow}, S)$ with $D^{\uparrow} = \bigcup_{S' \succeq S}(r^{S'}_S)^{-1}(D)$ as the upwards closure of $D \subseteq S \in \mathcal{S}$. Uncertainty and unawareness are both represented by a possibility correspondence Π_a for each $a \in Ag$, that maps a state weakly downwards to the set of states the agent considers possible. The possibility correspondence satisfies five properties (Confinement, Generalized Reflexivity, Stationarity, Projections Preserve Ignorance and Projections Preserve Knowledge), which capture HMS' intuitions on awareness and reflect standard assumptions on knowledge (see [2] for details).

An unawareness frame is then defined as the tuple $\mathsf{F} = (\mathcal{S}, \preceq, \mathcal{R}, \Pi)$, with set of events Σ_{F}. By adding a valuation $V_{\mathsf{M}} : At \longrightarrow \Sigma_{\mathsf{F}}$ to F, the HMS model is obtained, denoted $\mathsf{M} = (\mathcal{S}, \preceq, \mathcal{R}, \Pi, V_{\mathsf{M}})$ [8].

3 KRIPKE LATTICE MODELS

Kripke lattice models are constructed starting from standard Kripke models $\mathsf{K} = (W, R, V)$ defined for $At' \subseteq At$, where the information

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cell of $a \in Ag$ at $w \in W$ is $I_a(w) = \{v \in W : wR_a v\}$. We define the set of restrictions of K to $X \subseteq At$ and then produce a complete lattice of restricted models, by simply ordering them in accordance with subset inclusion of the atoms. The *restriction* of K to $X \subseteq At$ is defined as the Kripke model $\mathsf{K}_X = (W_X, R_X, V_X)$ for X where

$W_X = \{w_X : w \in W\}$ with w_X as (w, X),

$R_{Xa} = \{(w_X, v_X) : (w, v) \in R_a\}$ and

$V_X : X \longrightarrow \mathcal{P}(W_X)$, s.t. for all $p \in X, w_X \in V_X(p)$ iff $w \in V(p)$.

For $\mathcal{K}(\mathsf{K}) = \{\mathsf{K}_X\}_{X \subseteq At}$ the set of restrictions of K, let the *restriction lattice* of K be $(\mathcal{K}(\mathsf{K}), \trianglelefteq)$, where $\mathsf{K}_X \trianglelefteq \mathsf{K}_Y$ iff $X \subseteq Y$.

Restriction lattices are the frames of our models. They are informally interpreted as mapping states to alternates of themselves in less expressive models: if $Y \subseteq X \subseteq At$, then w_Y is the alternate of w_X formally described by the smaller vocabulary of atoms, Y.

The accessibility relations of each Kripke model in the lattice account for the epistemic dimension. For the awareness dimension, we add an *awareness map* $\pi_a : \Omega_{\mathsf{L}} \longrightarrow \Omega_{\mathsf{L}}$, with $\Omega_{\mathsf{L}} = \bigcup \mathcal{K}(\mathsf{K})$, for each $a \in Ag$. The π-map relates a world w_X down to $\pi_a(w_X) = w_Y$ for some $Y \subseteq X$. It satisfies three properties (Downwards Projections, Introspective Idempotence, and No Surprises), which ensure that π respects our intuitions on awareness (see [2] for details).

A Kripke lattice model $\mathbf{K} = (\mathcal{K}(\mathsf{K}), \trianglelefteq, \pi)$ is then defined as the restriction lattice $(\mathcal{K}(\mathsf{K}), \trianglelefteq)$ augmented with an awareness map π.

4 EQUIVALENCE BETWEEN MODELS

To show the equivalence between HMS models and Kripke lattices, we first define two transforms to move from HMS to Kripke lattice models and back. Then, we show that the transforms not only produce models of the correct class, but also preserve finer details, as any model and its transform satisfy the same formulas.

From HMS models to Kripke lattices. This transform requires a somewhat involved construction as it must tease apart unawareness and uncertainty from the possibility correspondences, and track the distribution of atoms and their relationship to awareness:

Let $\mathsf{M} = (\mathcal{S}, \preceq, \mathcal{R}, \Pi, V_{\mathsf{M}})$ be an HMS model with maximal state-space T. For any $O \subseteq \Omega_{\mathsf{M}} = \bigcup_{S \in \mathcal{S}} S$, let $At(O) = \{p \in At : O \subseteq V_M(p) \cup \neg V_M(p)\}$. The *L-transform* of M is $L(\mathsf{M}) = (\mathcal{K}(\mathsf{K}), \trianglelefteq, \pi)$ with the Kripke model $\mathsf{K} = (W, R, V)$ for At given by $W = T$; R maps all $a \in Ag$ to $R_a \subseteq W^2$, s.t. $(w, v) \in R_a$ iff $r^T_{S(\Pi_a(w))}(v) \in \Pi_a(w)$; $V : At \longrightarrow \mathcal{P}(W)$ with $V(p) \ni w$ iff $w \in V_{\mathsf{M}}(p)$, for all $p \in At$; and π assigns each $a \in Ag$ a map $\pi_a : \Omega_{L(\mathsf{M})} \longrightarrow \Omega_{L(\mathsf{M})}$ s.t. for all $w_X \in \Omega_{L(\mathsf{M})}$, $\pi_a(w_X) = w_Y$ where $Y = At(S_Y)$ for the $S_Y \in \mathcal{S}$ with $S_Y \supseteq \Pi_a(r^T_{S_X}(w))$, where $S_X = \min\{S \in \mathcal{S} : At(S) = X\}$.

Intuitively, in the L-transform model, a world $v \in W$ is accessible from a world $w \in W$ for an agent if, and only if, v's restriction to the agent's vocabulary at w is one of the possibilities she entertains. In addition, the awareness map π_a of agent a relates a world w_X to its less expressive counterpart w_Y if, and only if, Y is the vocabulary agent a adopts when describing what she considers possible.

As unawareness and uncertainty are separated in Kripke lattice models, we show two results. The first shows that for any HMS model M, its L-transform $L(\mathsf{M})$ is a Kripke lattice model, and the second that if $L(\mathsf{M}) = (\mathcal{K}((W, R, V), \trianglelefteq, \pi)$ is the L-transform of an HMS model M, then for every $a \in Ag$, R_a is an equivalence relation.

From Kripke lattices to HMS models. This transform requires a less involved construction, as the restriction lattice almost encodes projections, and unawareness and uncertainty are simply composed to form possibility correspondences:

Let $\mathbf{K} = (\mathcal{K}(\mathsf{K} = (W, R, V)), \trianglelefteq, \pi)$ be a Kripke lattice model for At. The *H-transform of* $\mathbf{K}$ is $H(\mathbf{K}) = (\mathcal{S}, \preceq, \mathcal{R}, \Pi, V_{H(\mathbf{K})})$ where $\mathcal{S} = \{W_X \subseteq \Omega_{\mathbf{K}} : \mathsf{K}_X \in \mathcal{K}(\mathsf{K})\}$; $W_X \preceq W_Y$ iff $\mathsf{K}_X \trianglelefteq \mathsf{K}_Y$; $\mathcal{R} = \{r^{W_X}_{W_Y} : r^{W_X}_{W_Y}(w_X) = w_Y$ for all $w \in W$, all $X, Y \subseteq At\}$; $\Pi = \{\Pi_a \in (2^{\Omega_{\mathsf{K}}})^{\Omega_{\mathsf{K}}} : \Pi_a(w_X) = I_a(\pi_a(w_X))$ for all $w \in W, X \subseteq At, a \in Ag\}$; $V_{H(\mathbf{K})}(p) = \{w_X \in \Omega_{\mathbf{K}} : X \ni p$ and $w_X \in V_X(p)\}$ for all $p \in At$.

As HMS models lump together unawareness and uncertainty, we show only one result in this direction, namely that for a Kripke lattice model $\mathbf{K} = (\mathcal{K}((W, R, V), \trianglelefteq, \pi)$ where R assigns equivalence relations, the H-transform $H(\mathbf{K})$ is an HMS model.

The equivalence of HMS and Kripke lattice models is shown with respect to the following language $\mathcal{L}$, with $a \in Ag$ and $p \in At$:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_a\varphi$$

In HMS settings, awareness $A_a\varphi$ is definable as $K_a\varphi \vee K_a\neg K_a\varphi$. The semantics for $\mathcal{L}$ over HMS or Kripke lattices are three-valued, as in each model it is possible that a state satisfies neither φ nor $\neg\varphi$. This happens when φ contains atoms not in X (see [2]).

We thus show that the L- and H-transforms produce models that are modally equivalent with respect to formulas in $\mathcal{L}$.

Lastly, HMS provide an axiom system for awareness and knowledge (see [8]). Call Λ_{HMS} the logic generated by such system and $\mathcal{M}$ the class of HMS models. HMS show that Λ_{HMS} is sound and strongly complete with respect to $\mathcal{M}$ [8]. Call $\mathcal{KLM}_{EQ}$ the class of Kripke lattice models with equivalence relations. As a corollary to the completeness of Λ_{HMS} with respect to $\mathcal{M}$ and our transformation and equivalence results, we obtain that Λ_{HMS} is sound and strongly complete with respect to $\mathcal{KLM}_{EQ}$.

5 DYNAMICS IN KRIPKE LATTICES

To model awareness dynamics, we apply a standard action model to the top model of the Kripke lattice, and compute the product update in the ordinary way [1]. We then spawn a Kripke lattice from the updated top model, and update the awareness map with all the atoms that are in the preconditions of all events in the action model and that are defined in the mapped-from state.

Let $\mathbf{K} = (\mathcal{K}(\mathsf{K} = (W, R, V)), \trianglelefteq, \pi)$ be a Kripke lattice model for At. Let $\mathcal{A} = (E, Q, pre)$ be a standard action model and let $At(pre)_X = \{p \in At : p$ is a subformula of $pre(e)$, for all $e \in E\} \cap X$, with $X \subseteq At$. The *awareness update* of $\mathbf{K}$ with $\mathcal{A}$ is $\mathbf{K}^{\mathcal{A}} = (\mathcal{K}(\mathsf{K}^{\mathcal{A}}), \trianglelefteq^{\mathcal{A}}, \pi^{\mathcal{A}})$ with $\mathsf{K}^{\mathcal{A}} = (W^{\mathcal{A}}, R^{\mathcal{A}}, V^{\mathcal{A}})$ for At given by $W^{\mathcal{A}} = \{(w, e) \in W \times E : M, w \vDash pre(e)\}$; $R^{\mathcal{A}}$ is s.t. $((w, e), (v, f)) \in R^{\mathcal{A}}_a$ iff $(w, v) \in R_a$ and $(e, f) \in Q_a$, for all $a \in Ag$; $V^{\mathcal{A}}(p) = \{(w, e) \in W^{\mathcal{A}} : w \in V(p)\}$ for all $p \in At$. Lastly, let $(\mathcal{K}(\mathsf{K}^{\mathcal{A}}), \trianglelefteq^{\mathcal{A}})$ be a restriction lattice, and $\pi^{\mathcal{A}}$ be s.t., for all $a \in Ag$, $\pi^{\mathcal{A}}_a((w, e)_X) = (w, e)_Z$ iff $Z = At(pre)_X \cup Y$ and $\pi_a(w_X) = w_Y$.

The choice to update agents' awareness with possibly all the atoms present in the preconditions of all events is suggested by the standard interpretation of epistemic and action models, where the whole model is common knowledge among the agents. Another option to model awareness dynamics in Kripke lattices is by defining *lattice* action models, which capture private awareness updates as well. Future research will also focus on making explicit the relation between ours and existing models for awareness dynamics [3–6, 10].

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