Democracy + Logic: Towards a General Approach to Conflict Resolution

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ABSTRACT

Some irreconcilable conflicts can be resolved by voting, which is part of the main technique of democracy. But democracy may fail, in the sense that voting by majority may not produce a rational result. The problem of democracy failure can be reformulated in the framework of judgment aggregation, so that it is turned into a purely logical problem—inconsistency resolution. By applying typical logical methods of inconsistency resolution, including belief merging, paraconsistent logic, and nonmonotonic logic, the problem can be finally resolved. We show how this 'democracy + logic' approach could be developed for conflict resolution.

KEYWORDS

Conflict Resolution, Judgment Aggregation, Belief Merging, Paraconsistent Logic, Nonmonotonic Logic

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1 INTRODUCTION

It is well known that democracy by voting is an important and general method for coping with conflicts among individuals and/or groups with different interests, preferences, opinions, and/or values, especially when their differences are irreconcilable. Democratic mechanisms (like voting by the majority rule), however, may fail to produce rational decisions. This is demonstrated by Arrow's famous impossibility theorem [1]. A half century after Arrow's work, [5] proposed the framework called judgment aggregation (a.k.a. logical aggregation [6]). It not only generalizes the framework of preference aggregation in social choice theory, but also turns the problem of practical rationality under democracy into a purely logical problem of inconsistency, for which extensive logical theories and methods are available.

This paper attempts to propose a general approach to conflict resolution. The approach is a combination of democracy and logic, which can be decomposed into three steps. First, we try the usual democratic methods (like voting by the majority rule). If they fail to come up with a rational decision, then we reformulate the problem in the framework of judgment aggregation and turn it into a logical problem of inconsistency. Finally, we use logical theories and methods for inconsistency resolution to solve the problem.

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2 JUDGMENT AGGREGATION: FROM CONFLICTS TO INCONSISTENCY

To show how the problem of practical conflict can be turned into the problem of logical inconsistency, let us review the following classical example motivating judgment aggregation. Consider a court with three judges. Suppose legal doctrine stipulates that the defendant is *liable* (r) iff there has been a valid *contract* (p) and that contract has been *breached* (q): $r \leftrightarrow p \wedge q$. The three judges' opinions are depicted in Table 1:

Table 1: The doctrine paradox

	p	q	r
Judge 1:	√	√	√
Judge 2:	√	×	×
Judge 3:	×	√	×
Majority:	√	√	×

Now we have a conflict in deciding whether to accept r. If we adopt a premise-based method, then by the majority rule, the collective would accept both p and q. Applying the legal provision $r \leftrightarrow p \wedge q$, the collective should accept r. If we adopt a conclusion-based method, then by the majority rule, the collective should reject r. This practical dilemma can be turned into a logical problem if we include the legal provision $r \leftrightarrow p \wedge q$ in judgments, as depicted in Table 2:

Table 2: The discursive dilemma

	p	q	r	$p \wedge q \leftrightarrow r$
Judge 1:	√	√	√	√
Judge 2:	√	×	×	√
Judge 3:	×	√	×	√
Majority:	√	√	×	√

By applying the majority rule on each proposition, the collective get an inconsistent set of judgments, though each set of individual judgments is consistent. This is called the discursive dilemma, whose generalization is one of the main topics of judgment aggregation (see [4] for an overview).

3 INCONSISTENCY RESOLUTION: VARIOUS LOGICAL APPROACHES

Since conflicts of preferences are turned into inconsistency of propositions in judgment aggregation, we can use various logical tools to solve the problem. We examine three methods for inconsistency resolution in turn.

3.1 Belief merging

In belief merging, the information sources need not be restricted by a given agenda. Moreover, distance-based aggregation is often used instead of proposition-wise aggregation. One possible method is by minimizing the distance from the collective belief base to the individual belief bases. Borrowing this idea, [7] gave a logical solution to the discursive dilemma. First, it defines the distance between a model satisfying some integrity constraints and an individual belief base. Then it defines the collective distance from those individual distances. Finally, it selects the (possibly more than one) base that minimizes the collective distance as the outcome of merging.

Though the distance-based procedure provides a systematic approach to resolving the inconsistency of collective judgment sets, there is a basic problem concerning the very idea of distances. In fact, there are two basic methods of defining distances between sets of judgments, which are seriously blurred in the literature.

Let $A\delta B = (A - B) \cup (B - A)$. Let $\mathcal{J}$ be the set of all judgment sets on a given agenda. We distinguish the two methods of defining distances as follows.

Definition 1. A distance metric $d : \mathcal{J} \times \mathcal{J} \to \mathbb{R}$ is *syntax-based* if $J_1\delta J_2 \subset J_3\delta J_4$ implies $d(J_1, J_2) < d(J_3, J_4)$; it is *model-based* if $\mathrm{Mod}(J_1)\delta\mathrm{Mod}(J_2) \subset \mathrm{Mod}(J_3)\delta\mathrm{Mod}(J_4)$ implies $d(J_1, J_2) < d(J_3, J_4)$ (assuming each J_i has only one model).

The two methods are not always compatible. We have the following result.

Proposition 2. *There is no distance metric that is both syntax-based and model-based for all judgment sets.*

The idea borrowed from belief merging in solving the discursive dilemma uses logic only implicitly. A direct use of logic we suggest in solving the inconsistency problem in judgment aggregation is paraconsistent logic.

3.2 Paraconsistent logic

Paraconsistent logic is a branch of non-classical logic that does not allow a pair of contradictory propositions to entail everything. It is usually assumed that the logic for collective rationality is the same as the that for individuals. The discursive dilemma, however, indicates that there may be different logics for the collective, as long as we take a more pluralistic view on logic. For example, if we allow the collective to accept both A and B without accepting $A \wedge B$, then the discursive dilemma is resolved. It is quite interesting that giving up the conjunction rule

$$\frac{A, B}{A \wedge B}$$

is one of the characteristics of the first paraconsistent logic, called discursive logic (a.k.a. discussive logic [2]). We suggest that this is far from a coincidence. The connection between discursive logic (and more generally, paraconsistent logic) and the discursive dilemma should be further explored.

But giving up the conjunction rule seems too radical. After all, as a very basic rule in classical logic, it is part of the very meaning of conjunction. Instead of giving it up completely, non-adjunctive systems suggested in [9] are less dramatic. The idea is that if the set of premises is inconsistent, then its maximal consistent subsets are used for derivation. More precisely, φ is a non-adjunctive consequence of Γ iff φ is a classical consequence of some maximal consistent subset of Γ. Then we have $\{p, q\} \vDash p \wedge q$ but $\{p, \neg p\} \nvDash p \wedge \neg p$. So the conjunction rule is partly retained whereas we still have the characteristic property $\{\varphi, \neg\varphi\} \nvDash \psi$.

The idea of using maximal consistent subsets leads to another branch of inconsistency resolution in logic: nonmonotonic logic.

3.3 Nonmonotonic logic

The theory of default reasoning due to [8] was one of the most famous theories in nonmonotonic logic. It provides another method for coping with inconsistency in judgment aggregation. Take the discursive dilemma as an example again. If we represent the three judges' opinions as default rules, then the three judges have the following default theories, respectively:

$$D_1 = (\emptyset, \{\frac{:p}{p}, \frac{:q}{q}, \frac{:r}{r}, \frac{:p \wedge q \leftrightarrow r}{p \wedge q \leftrightarrow r}\})$$
$$D_2 = (\emptyset, \{\frac{:p}{p}, \frac{:\neg q}{\neg q}, \frac{:\neg r}{\neg r}, \frac{:p \wedge q \leftrightarrow r}{p \wedge q \leftrightarrow r}\})$$
$$D_3 = (\emptyset, \{\frac{:\neg p}{\neg p}, \frac{:q}{q}, \frac{:\neg r}{\neg r}, \frac{:p \wedge q \leftrightarrow r}{p \wedge q \leftrightarrow r}\})$$

Using the majority rule on this profile of default theories, we get the collective default theory

$$D_m = (\emptyset, \{\frac{:p}{p}, \frac{:q}{q}, \frac{:\neg r}{\neg r}, \frac{:p \wedge q \leftrightarrow r}{p \wedge q \leftrightarrow r}\}).$$

By default logic, D_m has four (consistent) extensions

$$E_1 = Th\{p, q, p \wedge q \leftrightarrow r\}$$
$$E_2 = Th\{p, \neg r, p \wedge q \leftrightarrow r\}$$
$$E_3 = Th\{q, \neg r, p \wedge q \leftrightarrow r\}$$
$$E_4 = Th\{p, q, \neg r\}$$

This may not be acknowledged as a solution, since instead of a dilemma between two choices, we now have to decide among four choices. But this is not always the case. Recall that in default reasoning, we can reduce extensions by using priority over default rules.

In general, we will show that some common aggregation rules for evading the impossibility results can be transformed to a form of inconsistency resolution via nonmonotonic logic (c.f. [3]).

4 BEYOND JUDGMENT AGGREGATION

The above logical methods for inconsistency resolution indicate that the framework of judgment aggregation itself could be surpassed. First, individuals' opinions can be more flexibly represented than being restricted on a given agenda. The aggregation style can also go beyond proposition-wise, violating the independence condition. This is the lesson we learn from belief merging. Second, it is not necessary to stipulate that the collective use the same logic as individuals (c.f. [10]). For collective rationality, paraconsistent logic provides a promising alternative. Finally, opinions represented by default theories (and more generally in nonmonotonic logic) rather than sets of propositions are more general and inconsistency tolerable. This is the lesson we learn from nonmonotonic logic.

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