Some Dynamic Extensions of Social Epistemic Logic

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ABSTRACT

We propose two dynamic extensions to the framework of Social Epistemic Logic of [11]. One of them introduces an operator to break links within an epistemic social network based on information at the agent's disposal, whereas the other introduces a Twitter-like dynamic epistemic logic wherein the sending of a tweet by a user and its reading by the user's followers occur separately. Completeness results are provided.

KEYWORDS

Social Epistemic Logic, Public Announcements, Asynchronous Systems

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1 INTRODUCTION

This text aims to study a fundamental aspect of social networks (such as Facebook or Twitter) – namely, that of *change*. The change users effect on social networks, be it in the way of posting or establishing links with other users, is the reason we keep coming back to them. Conversely, these posts can change the users' epistemic state, their knowledge and beliefs, oftentimes to headline-worthy extents.

A multi-modal framework for reasoning about knowledge within networks of agents was introduced in [11], and iterated upon in multiple posterior works [9, 10, 12–14].

Section 2 introduces this framework (baptised in [14] as *Social Epistemic Logic*, or *SEL*), as well as some results about the logic it gives rise to.

SEL is a bidimensional framework wherein agents reason epistemically based on the information available in their cluster of epistemically indistinguishable worlds, worlds in which the pairs of agents connected by 'friendship' relations may vary. Dynamic extensions of SEL along its epistemic dimension have been studied in [11–13], among others. These texts use varied tools such as action models, PDL-style semantics, or AGM-style belief revision.

Section 3 proposes an extension along the other dimension, briefly exploring the relatively uncharted (with some exceptions in [12]) notion of a 'social update'. In this tentative proposal, we model the notion of an agent choosing to 'unfollow' a group of agents based on information about these users. This is based on Arrow Update Logics of [8].

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Finally, Section 4 addresses an arguably unrealistic assumption in the dynamic extensions carried over from Dynamic Epistemic Logic: namely, that epistemic change occurs immediately when a message is sent. Based on [5], we propose an *asynchronous* Social Epistemic Logic.

2 SOCIAL EPISTEMIC LOGIC

The SEL framework introduced in [11] utilises a bimodal logical language including an epistemic operator K for knowledge and a 'social' operator F to indicate such things as 'all my friends ϕ'. On top of this, and in order to name the agents in the network, it borrows some tools from Hybrid Logic [1, 6]: in addition to a countable set of propositional variables Prop, a countable set $\mathrm{Nom} = \{n, m, \dots\}$ of nominal variables is added to the language, which is also extended, for each $n \in \mathrm{Nom}$, with an atom n and a modal operator $@_n\phi$.

Models for SEL are tuples $(W, A, \sim, R, V)$ where W and A are nonempty sets (of 'worlds' and 'agents' respectively), $\sim = \{\sim_a\}_{a\in A}$ is a family of equivalence relations on W indexed by A (indicating indistinguishable worlds for agent a) and $R = \{R_w\}_{w\in W}$ is a family of 'friendship' relations on A indexed by W (indicating which pairs of agents are friends at world w), and V is a valuation such that $V(p) \subseteq W \times A$ for $p \in \mathrm{Prop}$ and $V(n) \in A$ for $n \in \mathrm{Nom}$. Assuming an agent 'is friends with herself', we demand reflexivity of the R_w relations.

Formulas in the above language are read with respect to pairs $(w, a) \in W \times A$ in these models as follows: $(w, a) \models K\phi$ iff $w \sim_a v$ implies $(v, a) \models \phi$; $(w, a) \models F\phi$ iff aR_wb implies $(w, b) \models \phi$; $(w, a) \models n$ iff $V(n) = a$; $(w, a) \models @_n\phi$ iff $(w, V(n)) \models \phi$.

A sound and complete axiomatisation of SEL is given in [10], and consists of the following axioms and rules:

(Taut)	all propositional tautologies	(MP)	Modus Ponens
($\mathrm{S5}_K$)	the S5 rules for the K operator	(Ref_F)	$\neg @_n F \neg n$
(K_F)	$F(\phi \to \psi) \to (F\phi \to F\psi)$	(Nec_F)	from ϕ, infer $F\phi$
($\mathrm{K}_@$)	$@_n(\phi \to \psi) \to (@_n\phi \to @_n\psi)$	($\mathrm{Nec}_@$)	from ϕ, infer $@_n\phi$
(Ref)	$@_n n$	(Selfdual)	$\neg @_n\phi \leftrightarrow @_n\neg\phi$
(Elim)	$@_n\phi \to (n \to \phi)$	(Agree)	$@_n @_m\phi \to @_m\phi$
(Back)	$@_n\phi \to F @_n\phi$	(DCom)	$@_n K @_n\phi \leftrightarrow @_n K\phi$
($\mathrm{Rigid}_=$)	$@_n m \to K @_n m$	($\mathrm{Rigid}_\neq$)	$\neg @_n m \to K \neg @_n m$

(Name) From $@_n\phi$ infer ϕ, where n is fresh in ϕ

(LBG) From $L(@_n \hat{F} m \to @_m\phi)$ infer $L(@_n F\phi)$, m fresh in $L(@_n F\phi)$, where $L(\#) ::= \# | \phi \to L | @_n K L$.

As shown in [2], SEL is decidable.

3 SOCIAL UPDATES IN SEL

We now introduce a notion of *social updates*, in order to model such things as 'I don't want to be Facebook friends with anyone I know to be a fascist', 'I don't wish to follow my ex-husband on Instagram', or more generally, the concept of 'breaking a friendship' or 'unfollowing a user' based on one's information in a social network. We do this by introducing an *arrow update operator*, based on [8]'s Arrow Update Logics.

Definition 1. *An* arrow update *is a finite set* $U = \{(n_1, \phi_1), \ldots, (n_k, \phi_k)\}$ *of pairs of nominal variables and formulas. For each such set we introduce an operator* $[U]\phi$*, interpreted as follows:*
$M, w, a \models [U]\phi$ *iff* $M^U, w, a \models \phi$*, where* $M^U = (W, A, \sim, R^U, V)$*, and* $R^U_w ab$ *iff* $R_w ab$ *and* $\exists(n, \psi) \in U : V(n) = a$ *&* $M, w, b \models \psi$*.*

Via an argument that involves 'reducing' a formula in the extended language to an equivalent one in the update-free fragment (see e.g. [7] for details on this proof method), one has:

Proposition 1. *The sound and complete logic of SEL with social updates is SEL plus the following reduction axioms and rule:*

$[U]p \leftrightarrow p$;	$[U]\neg\phi \leftrightarrow \neg[U]\phi$;
$[U](\phi_1 \wedge \phi_2) \leftrightarrow [U]\phi_1 \wedge [U]\phi_2$;	$[U]@_n\phi \leftrightarrow @_n[U]\phi$;
$[U]K\phi \leftrightarrow K[U]\phi$	$[U]F\phi \leftrightarrow \bigwedge_{(n,\psi)\in U} (n \to F(\psi \to [U]\phi))$

(Nec_U) From ϕ, infer $[U]\phi$.

4 ASYNCHRONICITY IN SOCIAL NETWORKS

Dynamic epistemic extensions of SEL suffer from an assumption carried over from Public Announcement Logic – namely the fact that an announcement is received by the agents at the very moment it is sent. This does not reflect the workings of a social network such as Twitter, wherein one agent might send a tweet at one point in time, and this tweet will be read by this agent's followers when they check their Twitter timelines hours or days later.

Basing it loosely on the Asynchronous Announcement Logics of [3–5], we briefly present a framework for asynchronous reception of tweets. Our language will now contain *sending* modalities $[n!\phi]$ and *reading* modalities $[n|\mathbf{r}]$, the former representing agent n sending a tweet and the latter agent n reading all tweets *sent by her Twitter friends* since the last time she checked Twitter.

To avoid circularity issues, an agent cannot use possible unread tweets to inform her reasoning. Since an agent remains ignorant about other agents' sent-but-unread tweets, and about who has read what in the network, she may never achieve certain knowledge of someone else's ignorance. For these (among other) reasons we use a doxastic modality B instead of the epistemic K. Moreover, we will demand (perhaps controversially) than an agent must believe the contents of a message to be true before tweeting it.

A *history* is a finite sequence of announcements and readings. E.g. $\alpha = n_1!p, n_2!\neg q, n_1|\mathbf{r}, n_1!Bp$ is a history. We abbreviate by $[\alpha]\phi$ the sequence of announcements and reading modalities represented by α. In the example, $[\alpha]\phi := [n_1!p][n_2!\neg q][n_1|\mathbf{r}][n_1!Bp]\phi$.

Given such a sequence of events α, the tweets an agent will read will depend on the occurrences of her name n in reading modalities and who her friends are at a world w. We thus need a specification of which nominals refer to different agents and which agents are friends in order to define this.

Definition 2. *Given a finite set of nominals* $N \subset \mathsf{Nom}$*, an* N-pseudomodel *is a triple* (A, w, a) *where* A *is a partition of* N*,* $a \in A$*, and* $w \subseteq A^2$ *is a reflexive binary relation.*

Given a history α *wherein only nominals from N occur,* $\alpha\restriction^A_{(w,a)}$ *is a finite sequence, whose length is exactly the number of readings* $n|\mathbf{r}$ *in* α *such that* $n \in a$*, whose elements are the sequences of announcements sent by 'friends' of* a *(according to the* w *relation) in between two consecutive such readings.*

For instance, if $\alpha = n_1!p, n_2!q, n_1|\mathbf{r}, n_1!q, n_3!(p \wedge q), n_1|\mathbf{r}$*, and* $A = \{a, b\}$ *with* $a = \{n_1, n_3\}, b = \{n_2\}$*, and* $w = \{(a, a), (b, b)\}$*, then* $\alpha\restriction^A_{(w,a)} = \langle (n_1!p), (n_1!q, n_3!(p \wedge q)) \rangle$*.*

A history β *is an* epistemic alternative to agent a in a (A, w, a)*, denoted* $\alpha \triangleright^A_{(w,a)} \beta$ *iff* $\alpha\restriction^A_{(w,a)} = \beta\restriction^A_{(w,a)}$ *and, moreover, no more announcements occur in* β *than those of* $\alpha\restriction^A_{(w,a)}$*.*

Given a SEL model $(W, A, \sim, R, V)$ *a history* α *whose nominals all occur in some* $N \subset \mathsf{Nom}$*, and* $(w, a) \in W \times A$*, we naturally obtain an* N*-pseudomodel* (A, w', a') *via the equivalence relation* $n \equiv m$ *iff* $V(n) = V(m)$*, with* $a' = V^{-1}(a) \cap N$*, and* $[n]w'[m]$ *iff* $V(n)R_wV(m)$*. The notions* $\alpha\restriction^M_{(w,a)}$ *and* $\alpha \triangleright^M_{(w,a)} \beta$ *are defined accordingly.*

We are now ready to define the semantics:

Definition 3. *We read formulas in SEL models with respect to triples consisting of a world* $w \in W$*, an agent* $a \in A$*, and a history* α *as follows:*

$w, a, \alpha \models [n!\phi]\psi$	*iff* $w, V(n), \alpha \models B\phi$ *implies* $w, a, \alpha \circ n!\phi \models \psi$*,*
$w, a, \alpha \models [n\vert\mathbf{r}]\phi$	*iff* $w, a, \alpha \circ n\vert\mathbf{r} \models \phi$*,*
$w, a, \alpha \models F\phi$	*iff* $R_w ab$ *implies* $w, b, \alpha \models \phi$*,*
$w, a, \alpha \models B\phi$	*iff* $v, a, \beta \models \phi$ *for all* v, β *such that* $w \sim_a v, \alpha \triangleright^M_{(w,a)} \beta$*, and* $v \bowtie \beta$*.*

The symbol $\circ$ *denotes concatenation; for the last line: a history* α *is* executable at world w*, denoted* $w \bowtie \alpha$*, iff either (i)* α *is the empty history, (ii)* $\alpha = \alpha' \circ n|\mathbf{r}$ *and* $w \bowtie \alpha'$*, or (iii)* $\alpha = \alpha' \circ n!\phi$*,* $w \bowtie \alpha'$ *and* $w, V(n), \alpha \models B\phi$*.*

For the logic of asynchronous SEL, we are interested in those formulas which hold everywhere with respect to the *empty history* $\varnothing$: we will say that ϕ is *valid* if, for every SEL model, and for every pair (w, a), it holds that $w, a, \varnothing \models \phi$.

Much like the framework in the preceding Section, this logic consists of the axioms and rules of SEL plus an assortment of reduction axioms. The most complex one is, as one might expect, the one having to do with the doxastic modality B.

Definition 4. *Given an* N*-pseudomodel* (A, w, a)*, we define the finite conjuction*

$$\mathsf{is}^A_{(w,a)} = \bigwedge_{n\in a} n \wedge \bigwedge_{n\in N\setminus a} \neg n \wedge \bigwedge_{n\equiv_A m} @_n m \wedge \bigwedge_{n\not\equiv_A m} \neg @_n m \wedge$$
$$\bigwedge_{\substack{n,m\in N \\ ([n],[m])\in w}} @_n \hat{F} m \wedge \bigwedge_{\substack{n,m\in N \\ ([n],[m])\notin w}} \neg @_n \hat{F} m,$$

This formula specifies which nominals of N refer to different agents and which pairs of agents are friends, and it is only true at a model if the N-pseudomodel induced by the model is precisely (A, w, a). Note that, for a finite N, there is a finite amount of N-pseudomodels, and that the disjunction of $\mathsf{is}^A_{(w,a)}$ over all of these will be true everywhere; more particularly, one exact disjunct thereof will be true at each pair (w', a') in a model.

Proposition 2. *Given a history* α*, let* N *be the set of nominals occurring in* α*. The following reduction is valid:*

$$[\alpha]B\phi \leftrightarrow [\alpha]\bot \vee \bigwedge_{\substack{(A,w,a) \\ N-pseudomodel}} \left(\mathsf{is}^A_{(w,a)} \to \bigwedge_{\alpha \triangleright^A_{(w,a)} \beta} B[\beta]\phi \right)$$

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