Epistemic Reasoning about Rationality and Bids in Auctions

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ABSTRACT

The goal of this work is to investigate strategic reasoning in the context of auctions. More precisely, we establish an explicit link between the agents' choice of bidding actions and bounded rationality. To do so, we extend the Auction Description Language with an epistemic operator and a choice operator and use it to represent a classical auction where agents have imperfect information about other bidders' valuations. We formalize bounded rationality concepts in iterative protocols and show how to use them to reason about the players' actions. Finally, we provide a model checking algorithm.

KEYWORDS

Logics for Multi-Agent Systems; Game Description Language; Bounded Rationality; Auction-Based Markets.

1 INTRODUCTION

Building a General Auction Player is similar to the General Game Playing challenge [5], it aims at designing an agent that can participate in an auction given the set of rules describing it. As for games, there is a wide variety of auction-based markets. Auctions may differ in the participants' type (e.g., only buyers, both buyers and sellers, ...), the kind and amount of goods being auctioned, the bidding protocol, and the allocation and payment rules [8].

Inspired by the Game Description Language (GDL), which is a logic programming language for representing and reasoning about game rules [5], we defined a general language to describe Auction-based markets from the auctioneer perspective [10]: *Auction Description Language* (ADL). In this paper we consider the player's perspective and our goal is to show how an agent may reason about the rules governing an Auction and also knowledge about other agents' valuation and behavior for eliciting her bid. More precisely, we show that computing a rational bid requires to assume that other agents are also bidding rationally. Following [2], we understand 'rational' as 'not playing dominated actions'.

Our contribution is twofold. We first extend ADL with knowledge operators from Epistemic GDL [7] and the action modality from the GDL variant proposed in [14]. Our goal is to provide the ground for the design of General Auction Players. Second, we characterize rationality along two dimensions: (i) the impact of the level of higher-order knowledge about other agents and (ii) the impact of looking-ahead beyond the next action to be executed. We also explore the model-checking complexity for evaluating rationality.

Related Work. To the best of our knowledge, there is no contribution that focuses on the strategic dimension of auctions through a logical perspective. However, numerous contributions define logical systems for representing games and representing strategic reasoning. General Game Playing uses the Game Description Language (GDL) [5] for representing games. The Auction Description Language (ADL) [10] extends GDL by handling numerical variables, a key feature for representing an Auction mechanism with its allocation and payment rules.

Alternating-time Temporal Logic (ATL) [1] provides a logic-based analysis of strategic decisions. Strategy Logic (SL) generalizes ATL with first-order quantifications over strategies [3]. These approaches cannot model the internal structures of strategies, which prevents to easily design strategies aiming to achieve a goal state. A logic for reasoning about composite strategies in turn-based games is introduced in [11], where strategies are treated as programs that are combined by PDL-like connectives. Zhang and Thielscher [15] present a variant of GDL to describe game strategies, where strategies can be understood as moves for a player. However, their work can only model turn-based games.

To incorporate imperfect information games, GDL has been extended to GDL-II [12] and GDL-III [13]. GDL-II and GDL-III aim at describing the rules of an imperfect information game, but do not provide tools for reasoning about how a player infers information based on these rules. All these logics face decidability and tractability issues: their expressive power prevents them from being implemented realistically in an artificial agent. Jiang et al. [7] propose an epistemic extension of GDL (EGDL) to represent and reason about imperfect information games. Their language allows us to represent the rules of an imperfect information game.

2 E-ADL LANGUAGE

The *Epistemic Auction Description Language* (E-ADL) is a framework based on ADL [10] and Epistemic GDL [6]. A formula in E-ADL, denoted $\varphi \in \mathcal{L}_{\text{E-ADL}}$, is defined by the following BNF:

$$\varphi ::= p \mid z < z \mid z > z \mid z = z \mid r \prec r \mid initial \mid terminal \mid legal(a^r) \mid does(a^r) \mid \neg\varphi \mid \varphi \wedge \varphi \mid \mathsf{K}_r\varphi \mid [\, d^{\mathrm{G}} \,]\varphi$$

where $p \in \Phi$ is a proposition, $r \in \mathrm{N}$ is an agent, $a^r \in \mathcal{A}$ is an action of agent r, $\mathrm{G} \in 2^{\mathrm{N}} \setminus \{\emptyset\}$ is a group of agents, $d^{\mathrm{G}} \in \prod_{r \in \mathrm{G}} \mathrm{A}^r$ is a joint action of the group G and $z \in \mathcal{L}_z$ is a numerical term.

Numerical terms in $\mathcal{L}_z$ represent integer values, mathematical functions (such as *sum* and *maximum*), and variables denoting numerical aspects from the auction (such as payments and allocations).

Intuitively, *initial* and *terminal* specify the initial and the terminal states, respectively; $does(a^r)$ and $legal(a^r)$ assert that agent r takes action a^r and that r is allowed to take action a^r at the current move, resp. The formula $\mathsf{K}_r\varphi$ is read as "agent r knows that φ". The action execution operator comes from the GDL variant with action modalities [14] and the formula $[\, d^{\mathrm{G}} \,]\varphi$ means that if joint action d^{G} is executed at the current state, φ will be true in all next moves.

The abbreviation $does(d^{\mathrm{G}})$ specifies that each agent in G performs her respective action in d^{G}. As in [14], we use the action modality to define the temporal operator $\bigcirc$, such that $\bigcirc\varphi$ means "φ holds next". Notation $\widehat{\mathrm{K}}_r$ stands for "φ is compatible with agent r's knowledge". Given $j > 0$ and $\mathrm{G} \in 2^{\mathrm{N}} \setminus \{\emptyset\}$, we write $\sigma^{\mathrm{G}} = (\prod_{r\in\mathrm{G}} \mathrm{A}^r)^j$ for a sequence of joint actions for G. The formula $[\,\sigma^{\mathrm{G}}\,]^j\,\varphi$ means that if the group G follows the sequence of joint actions described by σ^{G} for the next j stages, then φ will hold.

Similar to ADL [10] and Epistemic GDL [6], the semantics for E-ADL is given by a state transition (ST) models, which describes the states, the legality of actions, the valuation of variables and propositions, the accessibility relation for each agent, and the transition function. The truth value of a formula $\varphi \in \mathcal{L}_{\text{E-ADL}}$ is evaluated at a move (w, d) under an ST-Model M and denoted $\mathrm{M} \models_{(w,d)} \varphi$. A move is a pair of a state $w \in \mathrm{W}$ and a joint action $d \in \prod_{r\in\mathrm{N}} \mathrm{A}^r$.

Rationality in Auctions

The utility of an agent $r \in \mathrm{N}$ is denoted by the formula $util_r = u$, where u denotes how much utility the agent has at the current state (e.g. based on her current payment and whether she is the winner). We adapt the weak rationality formalization from [9] to E-ADL formulas. Different from his approach, we consider levels of rationality instead of common knowledge. Our notion of k-order rationality is based on [4]: an agent is k-order rational if she is weakly rational and knows all agents are $(k-1)$-order rational.

Since GDL-based agents chose "on-the-fly strategies" during the game, the players should be able to evaluate the current state of the game and to decide which action they will execute. For this reason, instead of defining utilities as a function to strategy profiles as in ATL [1], we model the agents' utility as dynamic: they depend on the current state of the auction.

We augment the rationality notions from [4, 9] by taking into account dynamic utilities. A rational agent plays according to the utility she will have after performing an action. When reasoning about iterative auctions, the agent considers the utility of playing according to a sequence of j actions. Since most auction-based markets are finite (in the sense that the auction finishes eventually), it is reasonable to assume the agents only need to include in their reasoning which actions may occur in the next j steps. Given a fixed number of steps $j > 0$, we inductively define that an agent is k-order rational, for $k \leq j$. The base case is that any agent is 0-order rational, that is $Rat(r, 0, j) \overset{\text{def}}{=} \top$. For all $k > 0$, we define:

$$Rat(r, k+1, j) \overset{\text{def}}{=} WRat(r, j) \wedge \mathrm{K}_r\Big(\bigwedge_{s\in\mathrm{N}} Rat(s, k, j)\Big)$$

That is, an agent is $k+1$-order rational if she is weakly rational when looking j stages ahead and knows every other agent is k rational. Weak rationality is defined by:

$$WRat(r, j) \overset{\text{def}}{=} \bigwedge_{a^r\in\mathrm{A}^r} \Big(does(a^r) \rightarrow \bigvee_{\sigma^r\in(\mathrm{A}^r)^{j-1}} WRat(r, (a^r, \sigma^r), j)\Big)$$

where

$$WRat(r, \sigma^r, j) =_{def} \bigwedge_{\chi^r\in(\mathrm{A}^r)^j} \Bigg(\bigvee_{\sigma^{\text{-}r}\in(\prod_{s\neq r}\mathrm{A}^s)^j} \Big(\widehat{\mathrm{K}}_r does(\sigma_1^{\text{-}r}) \wedge \bigvee_{u,u'\in\mathrm{V}} ([\,\chi^r, \sigma^{\text{-}r}\,]^j\, util_r = u' \wedge [\,\sigma^r, \sigma^{\text{-}r}\,]^j\, util_r = u \wedge u' \leq u)\Big)\Bigg)$$

An agent a^r is weakly rational when reasoning j stages ahead if when she performs an action a^r, there exists a sequence of j actions starting by a^r that is weakly rational for her to follow over j stages. Finally, it is weakly rational for agent r to follow a sequence of actions σ^r for j steps, noted $WRat(r, \sigma^r, j)$, if for every other sequence of actions χ^r there exists a sequence of joint actions $\sigma^{\text{-}r}$ that r considers possible to be executed such that her utility after following σ^r for j steps is at least as good as her utility after following σ^r.

First, notice that our definition of rationality requires to assume that all agents are rational: as soon as one is known to be non-rational, it is no longer possible to be k-order rational, for $k > 1$. Second, considering higher-order knowledge enables us to eliminate dominated actions.

Theorem 2.1. *For $k > 0$ and $j > 0$, for any ST-Model* M*, state $w \in \mathrm{W}$, joint action $d \in \prod_{r\in\mathrm{N}} \mathrm{A}^r$, agent r and action $a^r \in \mathrm{A}^r$,*

1. *If* $\mathrm{M} \models_{(w,d)} does(a^r) \wedge Rat(r, k, j)$ *then* $\mathrm{M} \models_{(w,d)} does(a^r) \wedge Rat(r, k-1, j)$
2. *If* $\mathrm{M} \not\models_{(w,d)} does(a^r) \wedge Rat(r, k-1, j)$ *then* $\mathrm{M} \not\models_{(w,d)} does(a^r) \wedge Rat(r, k, j)$

Note that increasing j may not enable the elimination of actions. The larger j, the more stages will be considered. Ideally, j should be large enough to reach terminal states. However, termination may not be ensured in auction protocols and real world players usually have time restrictions to decide their actions.

Model Checking

The complexity of the problem of deciding whether an E-ADL formula is true with respect to a model and a move is in exponential time in relation to the number of agents and the formula length. It follows that checking agent rationality is exponential in the number of agents, the order of rationality and number of rounds.

Theorem 2.2. *Given an ST-Model* M*, a state w, a joint action d and a formula φ, the problem of checking whether* $\mathrm{M} \models_{(w,d)} \varphi$ *or not is in* $O(|\mathrm{W}| \times |\mathcal{A}|^m)$*, where* $m = |\mathrm{N}| \times |\varphi|$.

Corollary 2.3. *Given an ST-model* M*, a state w, a joint action d, an agent r, $j > 0$ and $k > 0$, the problem of checking whether* $\mathrm{M} \models_{(w,d)} Rat(r, k+1, j)$ *is in* $O(|\mathrm{W}| \times |\mathcal{A}|^{nkj})$*, where* $n = |\mathrm{N}|$.

3 CONCLUSION

In this work, we present Epistemic Auction Description Language (E-ADL), a language to allow epistemic and action reasoning in auctions. E-ADL extends ADL with knowledge operators and action modalities from GDL variants. Our goal is to provide the ground for the design of General Auction Players and characterising their rational behavior. We also explore how to represent bounded rationality in relation to the level of higher-order knowledge about other agents and bounded looking-ahead beyond the next state. For future work, we intend to generalize the definitions to combinatorial auctions with complex valuation functions.

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