A Hennessy-Milner Theorem for ATL with Imperfect Information

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ABSTRACT

We show that a history-based variant of alternating bisimulation with imperfect information allows it to be related to a variant of Alternating-time Temporal Logic (ATL) with imperfect information by a full Hennessy-Milner theorem. The variant of ATL we consider has a *common knowledge* semantics, which requires that the uniform strategy available for a coalition to accomplish some goal must be common knowledge inside the coalition, while other semantic variants of ATL with imperfect information do not accomodate a Hennessy-Milner theorem. We also show that the existence of a history-based alternating bisimulation between two finite Concurrent Game Structures with imperfect information (iCGS) is undecidable.

CCS CONCEPTS

• **Theory of computation → Logic and verification**; • **Computing methodologies → Artificial intelligence**.

KEYWORDS

ATL, Concurrent Game Structures with Imperfect Information, Bisimulation, Gale-Stewart determinacy.

ACM Reference Format:
Francesco Belardinelli, Catalin Dima, Vadim Malvone, and Ferucio Tiplea. 2021. A Hennessy-Milner Theorem for ATL with Imperfect Information. In *ACM Conference, Washington, DC, USA, July 2017*, IFAAMAS, 2 pages.

1 INTRODUCTION

Alternating-time Temporal Logic (ATL) [3] is a powerful logic for specifying strategic abilities of individual agents and coalitions in multi-agent game structures. Crucially, ATL has been extended to games of imperfect information [17] with various flavors related to the agents' knowledge of the existence of strategies for accomplishing the coalition's goals [2, 8, 9]. In this contribution, we focus on the *common knowledge* (*ck*) interpretation of ATL under imperfect information, which was first put forward in [17], along with its *objective* and *subjective* interpretations. However, differently from the latter, to the best of our knowledge, the *ck* interpretation has nowhere else been considered in the literature. Nonetheless, the *ck* interpretation allows us to prove a Hennessy-Milner theorem for ATL under imperfect information for the memoryful notion of bisimulation we introduce in this paper. This result is in marked contrast with the situation for the other interpretations, which do not enjoy the Hennessy-Milner property [16].

The literature on bisimulations for modal logics is extensive, an in-depth survey of model equivalences for various temporal logics appears in [15]. The landscape for logics of strategic abilities, including ATL, is comparatively more sparse. A proof of the Hennessy-Milner property for ATL* with perfect information was already given in the paper introducing alternating bisimulations [4]. Since then, there have been numerous attempts to extend bisimulations to more expressive languages (including Strategy Logic recently [7]), as well as to contexts of imperfect information [1, 5, 10]. In [10, 18] non-local model equivalences for ATL with imperfect information have been put forward. However, these works do not deal with the imperfect information/perfect recall setting here considered, nor do they provide a local account of bisimulations. Further, in [5] the authors consider a *memoryless* notion of bisimulation for ATL, under imperfect information. Unfortunately, their definition does not allow for the Hennessy-Milner property. We also note the results from [11] which show that ATL with imperfect information is incompatible in expressive power when compared with the modal-epistemic μ-calculus, contrary to what is known for the perfect information case. The present contribution extends the notion of alternating bisimulation to the setting of imperfect information and perfect recall so that it satisfies the Hennessy-Milner property: two game structures are bisimilar iff they satisfy the same formulas in ATL.

The classic proof for Hennessy-Milner type properties typically uses bisimulation games played between Spoiler and Duplicator. These bisimulation games are turn-based, perfect information, safety games (in regards of Duplicator's objective) played on a tree whose nodes are labeled with pairs of states (or histories, in case of a memoryful semantics) of the two game structures checked for bisimulation. Hence, such games are determined, and determinacy plays a crucial role since, when there is no bisimulation between the two structures, the bisimulation game cannot be won by Duplicator, and hence Spoiler has a winning strategy, which is then used for exhibiting a formula that is satisfied in one structure but not in the other.

The extension of this proof technique to ATL with imperfect information has to cope with the fact that any notion of bisimulation has to account for the fact that coalitions have to choose action profiles in indistinguishable states in a "uniform" way: agents that do not distinguish between two states must choose the same actions in both. Uniformity entails a slightly more involved notion of bisimulation which utilizes *strategy simulators* [5]. Then, any bisimulation game has to encode these strategy simulators, in the sense that Duplicator is given the choice of a uniform strategy in some common-knowledge neighbourhood in one of the game structures and the Spoiler has to reply with a uniform strategy in the related common-knowledge neighbourhood of the other game structure.

The problem raised by this generalization is that positions in a bisimulation game are normally labeled with histories, not common-knowledge neighbourhoods, as bisimulations relate the former, not the latter. So, we need both a Spoiler and a Duplicator who have imperfect information at each position of the bisimulation game. On the other hand, as it is the case with bisimulations for the perfect information case, for each choice of strategies in the two structures, the outcomes of one strategy have to be related with the outcomes of the other strategy. But this requires both Spoiler and Duplicator to be *perfectly-informed*!

The solution we propose is a 4-player bisimulation game played between the Spoiler coalition {I-Spoil, P-Spoil} and the Duplicator coalition {I-Dupl, P-Dupl}, where both I-players have imperfect information, while both P-players have perfect information. We show that such a game can be won by the Duplicator coalition if and only if there exists a bisimulation between the two game structures.

Further, we provide a Gale-Stewart type determinacy theorem [14] for the bisimulation game, showing that exactly one of the two coalitions has a winning joint strategy. The key point is that, when Duplicator does not have a winning strategy, the strategic choices for I-Spoil can be defined in a uniform way that is only based on I-Spoil's observations. To the best of our knowledge, this is the first example of a class of multi-player, imperfect information, zero-sum (reachability) games played over infinite trees that are determined. Note that, for technical reasons, our Hennessy-Milner theorem only holds for ATL with the "yesterday" modality **Y**.

Moreover, we analyse the problem of checking the existence of a bisimulation between two given game structures. We show that this problem is undecidable in general by building on the undecidability of the model-checking problem for ATL with imperfect information and perfect recall. More specifically, given a Turing machine M, we build a game structure in which a two-agent coalition has a strategy for avoiding an error state if and only if M halts when starting with an empty tape. We then build a second, unrelated, simple game structure in which the same coalition always has an avoiding strategy. Finally, we prove that the two structures are bisimilar if and only if M halts.

We note that our Hennessy-Milner theorem utilizes the "yesterday" modality for technical reasons. The translation of this theorem to the full ATL* is left for future research.

As another direction for future research, we plan to investigate under which conditions our Gale-Stewart-type theorem can be generalized to a full determinacy theorem for multi-agent games.

Type of submission. The current submission is a short abstract of the following paper:

- Francesco Belardinelli, Catalin Dima, Vadim Malvone, Ferucio Laurentiu Tiplea: A Hennessy-Milner Theorem for ATL with Imperfect Information. *Proceedings of LICS 2020*, p.181–194.

ACKNOWLEDGMENTS

F. Belardinelli acknowledges the support of ANR JCJC Project SVeDaS (ANR-16-CE40-0021).

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