How Groups Can Help Coalitions

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ABSTRACT

In this extended abstract we present the themes and results of a recent paper [11][1]. We omit many definitions in order to make the abstract more readable, and for technicalities and pointers to the literature the reader is invited to consult the full paper. Moreover, in order to prevent excessive citing of [11], we refer to it as 'the paper'.

KEYWORDS

Coalition Announcement Logic; Group Announcement Logic; Public Announcement Logic; Coalition Logic

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1 INTRODUCTION

One way to extend various dynamic epistemic logics (DELs) [8] is to add quantification over corresponding epistemic actions (see [7] for a recent survey). Such an extension allows us to reason whether there is an action or a sequence of actions that will take us from some starting to some target configuration (note that this reasoning echoes the problem of epistemic planning [6]). The most studied of these extensions are variants of public announcement logic (PAL) with various types of quantification over announcements, with the most notable of them being presented in the big trio of arbitrary public announcement logic (APAL), group announcement logic (GAL), and coalition announcement logic (CAL).

In APAL, constructs $\Diamond\varphi$ intuitively mean that '*there is* an announcement such that φ is true after it'. Note that the quantification in APAL does not take into account who makes an announcement or whether the announcement can be made by any of the agents in the system. Hence, in GAL constructs $\langle G\rangle\varphi$ quantify over announcements known to agents from a group of agents G; formula $\langle G\rangle\varphi$ means that '*there is* an announcement by agents from G such that φ is true after it'.

Modalities $\langle G\rangle\varphi$ of GAL have a strategic flavour to them: given an initial configuration of a model, is there an action (announcement in our case) by a group of agents such that the model reaches a desired configuration (expressed by φ)? This intuition is further fleshed out in CAL, where constructs $\langle\!\![G]\!\!\rangle\varphi$ mean that '*there is* an announcement by agents from G such that *whatever* agents outside of the group announce at the same time, φ will be true after their joint announcement'. Modalities of CAL were motivated by coalition logic [13] and the playability operator [4]. Observe that the operators of CAL capture the property of α-effectivity, and thus the logic serves as a meeting point between DEL and game theory.

The reader may have noticed that APAL and GAL are quite similar. Indeed, they shared their highs and lows: similar infinitary axiomatisations and completeness proofs are offset with the unsoundness of their finitary axiomatisations. CAL is quite different from both APAL and GAL due to the fact that the quantification in CAL is of the form $\exists\forall$, and in APAL and GAL it is of the form $\exists$.

In what follows, we present the results of our investigation of CAL through the lens of GAL. In particular, we show some logical properties of the logics in parallel. Then, we introduce *relativiesed group announcements* $[G,\psi]\varphi$ and $\langle G,\psi\rangle\varphi$ that generalise classic group announcements and serve the essential role in the axiomatisation and the completeness of a logic with coalition announcements. Specifically, they split the $\exists\forall$ quantification in $\langle\!\![G]\!\!\rangle\varphi$. We finish with an observation on the strategic nature of announcements in CAL.

2 GROUPS AND COALITIONS

Let A be a finite set of agents, and P be a countable set of propositional variables. *Languages of group announcement logic and coalition announcement logic* are defined by the following BNF:

$$\mathbf{L} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_a\varphi \mid \langle\varphi\rangle\varphi \mid \langle G\rangle\varphi \mid \langle\!\![G]\!\!\rangle\varphi$$

where $p \in P$, $a \in A$, $G \subseteq A$, and all the usual abbreviations of propositional logic, conventions for deleting parentheses, and definitions of duals hold. Language $\mathbf{L}$ without $\langle\!\![G]\!\!\rangle\varphi$ is **GAL**, and $\mathbf{L}$ without $\langle G\rangle\varphi$ is **CAL**.

Formulas of (almost) all logics we are dealing with are interpreted in epistemic models. An *epistemic model* is a triple $M = (S, \sim, V)$, where $S \neq \emptyset$ is a set of states, $\sim\colon A \to 2^{S\times S}$ is an equivalence relation, $V : P \to 2^S$ is the valuation function. An *updated model* M^φ is $(S^\varphi, \sim^\varphi, V^\varphi)$, where $S^\varphi = \{s \in S \mid M_s \models \varphi\}$ ($\models$ is defined below), $\sim_a^\varphi = \sim_a \cap (S^\varphi \times S^\varphi)$ for all $a \in A$, and $V^\varphi(p) = V(p) \cap S^\varphi$.

Let $\mathbf{PAL}^G = \{\bigwedge_{i\in G} K_i\psi_i \mid \text{for all } i \in G, \psi_i \in \mathbf{PAL}\}$. The *semantics* is defined as follows (omitting propositional, boolean, and epistemic cases):

$$\begin{array}{lll} M_s \models \langle\psi\rangle\varphi & \text{iff} & M_s \models \psi \text{ and } M_s^\psi \models \varphi \\ M_s \models \langle G\rangle\varphi & \text{iff} & \exists\psi_G \in \mathbf{PAL}^G : M_s \models \langle\psi_G\rangle\varphi \\ M_s \models \langle\!\![G]\!\!\rangle\varphi & \text{iff} & \exists\psi_G \in \mathbf{PAL}^G, \forall\chi_{A\setminus G} \in \mathbf{PAL}^{A\setminus G} : \\ & & M_s \models \psi_G \text{ and } M_s \models [\psi_G \wedge \chi_{A\setminus G}]\varphi \end{array}$$

To avoid circularity, quantification in the definition of the semantics of $\langle G\rangle$ and $\langle\!\![G]\!\!\rangle$ is restricted to PAL formulas known to agents. Formula φ is *valid* iff for all M_s, $M_s \models \varphi$.

To draw some parallels between GAL and CAL, let us consider a selection of properties that are *not* valid. We write $\nrightarrow$ to stress that the implications are not valid.

[1] If the reader cannot access the paper due to the publisher's paywall, they are welcome to have a look at the author's final draft available at `https://rgalimullin.gitlab.io/JOLLI21/paper.pdf`

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- $\langle G \cup H\rangle\varphi \nrightarrow \langle G\rangle\langle H\rangle\varphi$ and $\langle\!\langle G \cup H\rangle\!\rangle\varphi \nrightarrow \langle\!\langle G\rangle\!\rangle\langle\!\langle H\rangle\!\rangle\varphi$. Informally this means that groups and coalitions may loose their joint strategies once separated.
- $\langle G\rangle[H]\varphi \nrightarrow [H]\langle G\rangle\varphi$ and $\langle\!\langle G\rangle\!\rangle[\![H]\!]\varphi \nrightarrow [\![\langle H\rangle]\!]\langle\!\langle G\rangle\!\rangle\varphi$. The generalised Church-Rosser [5] principle does not hold for neither GAL nor CAL, even though it holds for APAL [2].
- $\langle G\rangle[A \setminus G]\varphi \nrightarrow \langle\!\langle G\rangle\!\rangle\varphi$. This implies that an intuitive translation of CAL into GAL via $\langle G\rangle[A \setminus G]\varphi \leftrightarrow \langle\!\langle G\rangle\!\rangle\varphi$ does not work [9].

3 RELATIVISED GROUP ANNOUNCEMENTS

A sound and complete axiomatisation of CAL is still an open problem, and the inherent alternation of quantifiers in the semantics of $\langle\!\langle G\rangle\!\rangle\varphi$ is one of the main reasons why it is so. Moreover, the axiomatisation of any logic that includes coalition announcement modalities used to be an open question. We employ relativised group announcements to provide such an axiomatisation.

Relativised group announcement $\langle G, \psi\rangle\varphi$ means that 'given an announcement ψ, *there is* a *simultaneous* announcement by agents from G, such that φ will hold in the resulting model'. In other words, $\langle G, \psi\rangle\varphi$ is similar to $\langle G\rangle\varphi$ with the difference that to the formula that G announces we add ψ as a conjunct.

Formally, the semantics of $\langle G, \psi\rangle\varphi$ is as follows (we also add an alternative equivalent semantics of $\langle\!\langle G\rangle\!\rangle\varphi$):

$M_s \models \langle G, \chi\rangle\varphi$ iff $M_s \models \chi \Rightarrow \exists\psi_G \in \mathbf{PAL}^G : M_s \models \langle\psi_G \wedge \chi\rangle\varphi$

$M_s \models \langle\!\langle G\rangle\!\rangle\varphi$ iff $\exists\psi_G \in \mathbf{PAL}^G : M_s \models [A \setminus G, \psi_G]\varphi$

It is immediate that we can express normal group announcements and public announcements via relativised group announcements: $\langle G\rangle\varphi \leftrightarrow \langle G, \top\rangle\varphi$ and $\langle\psi\rangle\varphi \leftrightarrow [\emptyset, \psi]\varphi$.

The main motivation behind relativised group announcements is that they allow to keep one announcement in 'memory', and thus we can consider a coalition's announcement and the anti-coalition's response separately. This fact was also used in the expressivity result of [9], where there was a need to separate moves of players in a game that correspond to coalitions and anti-coalitions.

CAL with added relativised group announcements is shown to be complete in [10]. The axiom system of the resulting logic, *coalition and relativised group announcement logic* (CoRGAL) is an extension of the axiomatisation of PAL [14] with two new axiom schemas and two rules of inference

- $[G, \chi]\varphi \to \chi \wedge [\psi_G \wedge \chi]\varphi$ for any ψ_G,
- $[\![G]\!]\varphi \to \langle A \setminus G, \psi_G\rangle\varphi$ for any ψ_G,
- From $\{\eta(\chi \wedge [\psi_G \wedge \chi]\varphi) \mid \psi_G \in \mathbf{PAL}^G\}$, infer $\eta([G, \chi]\varphi)$,
- From $\{\eta(\langle A \setminus G, \psi_G\rangle\varphi) \mid \psi_G \in \mathbf{PAL}^G\}$, infer $\eta([\![G]\!]\varphi)$.

The completeness proof in [10] followed the now classical proof from [1, 3]. In the paper, however, we presented an alternative axiomatisation and an alternative proof. As a basis we took a variant of PAL presented in [15], where the authors introduced an axiomatisation of PAL that does not rely on reduction axioms. The main idea of the alternative PAL is to treat public announcement not as model-changing operators but as *static* relations akin to any other standard box modality.

This approach to public announcements demanded a new notion of an epistemic model, which was called *extended epistemic model* (abbreviated as EEM and denoted by $\mathcal{M}$), and where, in addition to the standard elements of a model, we have $\xrightarrow{\psi}$-transition for each

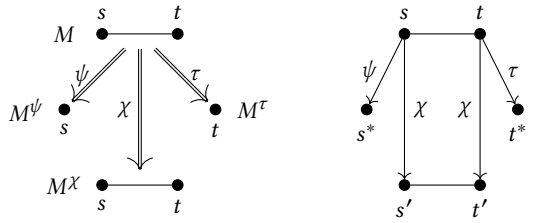

Figure 1: On the left: epistemic model M and three possible updates thereof, M^ψ, M^χ, and M^τ. On the right: extended epistemic model $\mathcal{M}$.

$\psi \in$ **CoRGAL** from each state. These new transitions model the effects of a public announcement of the corresponding formula. See Figure 1 for the illustration of the basic idea. The extended semantics for public announcements on EEM $\mathcal{M}_s$ is now as follows:

$$\mathcal{M}_s \models \langle\psi\rangle\varphi \quad \text{iff} \quad \text{there is } t \in S : s \xrightarrow{\psi} t \text{ and } \mathcal{M}_t \models \varphi$$

As the extended semantics suggests, public announcement of ψ has been equivalently replaced by a transition along $\xrightarrow{\psi}$. Observe that not every EEM has an equivalent epistemic model. To make it the case, the class of EEMs is restricted to those that respect perfect recall, no miracles, and other properties.

In the paper, we prove the completeness of the axiom system of CoRGAL, which is based on PAL without reduction axioms, via a detour through the restricted class of EEMs. For this, we combine proof techniques from [3] and [15]. The axiomatisation and completeness of *relativised group announcement logic* (without coalition announcements) follows straightforwardly.

4 PUBLIC ANNOUNCEMENTS AS STRATEGIES

Instead of a conclusion, we consider an interesting corollary result that we did not discuss in the paper. Let us have another look at EEMs. If we restrict ψ-transitions to only those that represent an announcement by some coalition G, then the resulting model will be almost an *epistemic concurrent game model* (CGM) (see, for example, [12]). We say 'almost' because in addition, we should also ensure that every ψ-transition is labelled with a *complete strategic profile*, i.e. for each agent $i \in A$ there is an action act_i in that labelling. However, in the case of announcements, each coalition announcement ψ_G can be equivalently rewritten as $\psi_G \wedge \top_{A\setminus G}$. Also note that the restrictions on EEMs (perfect recall et al.) remain.

Now, if $\mathcal{M}_s$ is an epistemic CGM thus obtained, the semantics of the coalition announcement modality $\langle\!\langle G\rangle\!\rangle\varphi$, coincide with the semantics of the classic coalition modality $\langle\langle G\rangle\rangle\varphi$:

$$\mathcal{M}_s \models \langle\!\langle G\rangle\!\rangle\varphi \quad \text{iff} \quad \exists\psi_G, \forall\chi_{A\setminus G} : \mathcal{M}_t \models \varphi, \text{ where } s \xrightarrow{\psi_G \wedge \chi_{A\setminus G}} t$$

Thus, the resulting logic, CAL without public announcements on restricted epistemic CGMs, is exactly the classic (epistemic) CL, where agents' strategies are public announcements. We have made the full circle from 'CAL being a DEL inspired by CL' to 'CAL being a variant of CL'. To the best of our knowledge, this strategic approach to the logics of quantified announcements has not been undertaken yet, and it is an intriguing alternative to the dominant DEL treatment.

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